# Redefining the Derivative

Previously, the derivative was defined at a fixed number :

Now, the derivative can be calculated at any point :

Graphically, **the derivative of a function** at any point is the **slope of the function** at that point.

# Derivative Notation

If , **the derivative of**  can be expressed in some ways (which all equal each other):

Differentiation – the operation of calculating the derivative.

Differentiation operators – symbols that indicate differentiation, like , , and .

**Note**: and other differentiation operators in a fraction are **just symbols, not ratios**. You can’t cancel out the ’s. (But later we’ll see how to combine derivatives with other, different derivatives.)

Using this notation, a derivative, or the **instantaneous rate of change**, Leibniz notation defines it as:

To express the derivative at an exact point, another Leibniz notation gives:

 or or

# Differentiable Functions

A function is **differentiable at**  if  **exists**.

A function is **differentiable on an interval**  if  **exists at every number in the interval**.

**Theorem & Proof.** If a function is **differentiable** at , it is **continuous** at .

**Remember,** a function continuous at **exists** at a and is **equal to its limit** when evaluated at .

We will try to prove that is continuous at a, that is, .

Expand .

Move the left denominator to the other side.

Evaluate the rightmost limit.

Multiply by zero.

Break apart the subtraction and move the minuend to the right side.

Evaluate the limit.

Therefore, is continuous at .

# Functions not Differentiable

Some functions have points or intervals that **don’t have a derivative** at those points or intervals. This means they are **not differentiable** at those points or intervals.

At a **corner** of a function, a point where the slope changes instantly, the derivative does not exist.

When a function **jumps**, it is not continuous. Therefore, the function is not differentiable at that point.

Since derivatives are defined as limits, if the derivative/slope/**tangent** at a point is **vertical** (infinity – what we used to call ‘does not exist’), the derivative doesn’t exist at that point.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| A corner | A jump | A vertical tangent |

# Higher Derivatives

Second Derivative – The derivative of a derivative.

Suppose ( or ) is differentiated across . We get:

A third derivative (derivative of the second derivative) would be written . Any higher derivative would be written in superscript and parentheses; for example, the fourth derivative of is .

Velocity () measures the rate of change of speed () over time ().

Acceleration () measures the rate of change of velocity () over time ().